Lesson 2 Factorial Notation Worksheet

1. Evaluate the following only using a calculator for the final step:

a. $\frac{8!}{5!}$

$$\boxed{\begin{array}{l}\frac{8\times7\times6\times5!}{5!}\\ =8\times7\times6=336\end{array}}$$

c. $\frac{0!}{4!}$

$$\boxed{\frac{1}{4\times3\times2\times1}=\frac{1}{24}}$$

b. $\frac{3!}{5!}$

$$\boxed{\begin{array}{l}\frac{3!}{5\times4\times3!}=\frac{1}{5\times4}\\ =\frac{1}{20}\end{array}}$$

d. $\frac{37!}{33!4!}$

$$\boxed{\begin{array}{l}\frac{37\times36\times35\times34\times33!}{33!4!}\\ \frac{37\times3\times35\times17}{4!}\\ \frac{{}_{37}P_4}{4!}=66\ 045\\ \text{calculator}\end{array}}$$

2. Simplify the following expressions:

a. $\frac{n!}{n}$

$$\boxed{\frac{n(n-1)!}{n}=(n-1)!}$$

d. $\frac{(3n)!}{(3n-2)!}$

$$\boxed{\begin{array}{l}\frac{(3n)(3n-1)(3n-2)!}{(3n-2)!}\\ 3n(3n-1)\end{array}}$$

b. $\frac{(n+1)!}{(n-1)!}$

$$\boxed{\frac{(n+1)n(n-1)!}{(n-1)!}=n(n+1)}$$

e. $\frac{n!(n+1)!}{(n-1)!(n-1)!}$

$$\boxed{\begin{array}{l}\frac{n\cancel{(n-1)!}\,(n+1)n\cancel{(n-1)!}}{\cancel{(n-1)!}\,\cancel{(n-1)!}}\\ n^2(n+1)\end{array}}$$

c. $\frac{(n-3)!}{(n-2)!}$

$$\boxed{\frac{(n-3)!}{(n-2)(n-3)!}=\frac{1}{n-2}}$$

3. Solve for n in the following expressions:

a. $\frac{(n+1)!}{n!}=6$

$$\frac{(n+1)n!}{n!}=6$$
$$n+1=6$$
$$n=5$$

d. $\frac{(n+2)!}{n!}=42$

$$\frac{(n+2)(n+1)n!}{n!}=42$$
$$n^2+3n+2=42$$
$$n^2+3n-40=0$$
$$(n+8)(n-5)=0$$
$$n=\cancel{-8},5$$

b. $(n+1)!=6(n-1)!$

$$\frac{(n+1)!}{(n-1)!}=6$$
$$\frac{(n+1)n(n-1)!}{(n-1)!}=6$$
$$n^2+n=6$$
$$n^2+n-6=0$$
$$(n+3)(n-2)=0$$
$$n=\cancel{-3},2$$

e. $\frac{(n+1)!}{(n-2)!}=20(n-1)$

$$\frac{(n+1)(n)(n-1)(n-2)!}{n-2!(n-1)}=20$$
$$n^2+n-20=0$$
$$(n+5)(n-4)=20$$
$$n=\cancel{-5},4$$

c. $\frac{(n-15)!}{(n-14)!}=\frac{1}{12}$

$$\frac{(n-15)!}{(n-14)(n-15)!}=\frac{1}{12}$$
$$\frac{1}{n-14}=\frac{1}{12}$$
$$n-14=12$$
$$n=26$$

f. $\frac{n!}{(n+2)!}=\frac{1}{6n}$

$$\frac{n!}{(n+2)(n+1)n!}=\frac{1}{6n}$$
$$\frac{1}{n^2+3n+2}=\frac{1}{6n}$$
$$n^2+3n+2=6n$$
$$n^2-3n+2=0$$
$$(n-2)(n-1)=0$$
$$n=1,2$$

4. How many arrangements are there of the letters.

a. DOG

$3!$ or ${}_3P_3$

b. SANDWICH

$8!$ or ${}_8P_8$

5. How many five-digit numbers can be made from the digits 2, 3, 4, 7 and 9 if no digit can be repeated?

$$\boxed{5! \text{ or } {}_5P_5}$$

6. If ${}_nP_r$ is the number of ways that n objects can be arranged r at a time, explain why ${}_7P_0 = 1$.

$$\boxed{{}_7P_0 = \frac{7!}{(7-0)!} = \frac{7!}{7!} = 1}$$

7. Use a permutation formula to determine how many arrangements there are of
   a. Two letters from the word GOLDEN

$$\boxed{{}_6P_2 \Rightarrow \frac{6!}{4!} = \frac{6\times5\times4!}{4!} = 30}$$

   b. Three letters from the word CHAPTERS

$$\boxed{\begin{array}{l} {}_8P_3 = \dfrac{8!}{5!} = \dfrac{8\times7\times6\times5!}{5!} \\ 8\times7\times6 = 336 \end{array}}$$

   c. Four letters from the word WEALTH

$$\boxed{\begin{array}{l} {}_6P_4 = \dfrac{6!}{2!} = \dfrac{6\times5\times4\times3\times2!}{2!} \\ = 360 \end{array}}$$

8. Solve each equation, where n is an integer:
   a. $\dfrac{n!}{84} = {}_{n-2}P_{n-4}$
   b. ${}_nP_4 = 8\left({}_{n-1}P_3\right)$

$$\boxed{\begin{array}{l} \dfrac{n!}{84} = \dfrac{(n-2)!}{\left[(n-2)-(n-4)\right]} \\ \dfrac{n!}{84} = \dfrac{(n-2)!}{2!} \\ \dfrac{1}{42} = \dfrac{\cancel{(n-2)!}}{n(n-1)\cancel{(n-2)!}} \\ n^2 - n - 42 = 0 \\ (n-7)(n+6) = 0 \\ n = 7 \end{array}}$$

$$\boxed{\begin{array}{ll} \dfrac{n!}{(n-4)!} = \dfrac{8(n-1)!}{(n-1-3)!} & \\ \dfrac{n!}{(n-4)!} = \dfrac{8(n-1)!}{(n-4)!} & \dfrac{n(n-1)!}{(n-1)!} = 8 \\ \dfrac{n!}{(n-1)!} = 8 & n = 8 \end{array}}$$